Qualitative Comparison of Reconstruction Algorithms for Diffusion Imaging

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**Introduction**

Diffusion Imaging is able to represent neural structures and is therefore becoming increasingly important for neuroimaging. It is based on the fact that diffusion anisotropy in white matter correlates to the direction of neural pathways. This diffusion can be determined with different reconstruction methods.

Diffusion imaging was firstly introduced by Basser et al. [1] in 1994. Since then, various reconstruction algorithms have been developed and proposed. Each of them aims as reconstructing the underlying diffusion or fiber structure. However, every reconstruction approach has its specific advantages and disadvantages. In this chapter an overview over different well-established basic reconstruction algorithms is presented and their differences are discussed.

In the following, diffusion imaging and its components are first introduced in Section 1, followed by well-established reconstruction algorithms in Section 2. Afterwards, Section 3 provides a qualitative comparison and a user guideline is presented.

**Diffusion Imaging**

Diffusion imaging is based on magnetic resonance imaging (MRI) and measures the diffusion of neural tissue. Based on the fact that the diffusion along white matter fibers is anisotropic, the direction of neural pathways can be estimated.

Using the Stejskal-Tanner equation [2], a diffusion coefficient can be measured using a directional gradient strength $|q| = \gamma \delta G$, where $\gamma$ is the gyromagnetic ratio, $\delta$ is the gradient pulse duration time and $G$ is the diffusion gradient vector. The gradient $G$ is defined by its length $b$ and its gradient direction $u$, where $b$ is proportional to the diffusion strength.

To reveal the underlying fiber orientation, every voxel has to be measured with different directions gradients $G$. Each applied gradient is assigned to a unique coordinate in the three-dimensional so called q-space, according to its length and direction. An illustration of different q-space acquisition schemes is presented in Figure 2. In general, the reconstruction accuracy increases with the number of measured gradient directions.
The q-space is similar to the k-space [3] in MRI, i.e. that it can be transformed via Inverse-Fourier-Transform (IFT) to reveal the orientation distribution function (ODF) of a voxel, which represents the diffusion profile in the measured voxel.

Considering the exemplary signal in Figure 2, it can be seen that the signal has low values in direction of a fiber (red and blue lines). This is due to the basic diffusion MRI principle, which measures the response signal of a previously emitted signal. If there is a high diffusion, only a weak signal is returned.

To recover the ODF (see Figure 2) a reconstruction algorithm is applied. While most reconstruction algorithms model the diffusion profile, some also directly reconstruct the direction of fibers. For this reason there are two types of ODFs: The First one represents the diffusion orientation distribution function (dODF) of water molecules, while the second one is denoted as fiber orientation distribution function (fODF). The fODF is similar to a sharpened version of the dODF.
Figure 2: Picture of different exemplary acquisition schemes (green: inner, red: middle, blue: outer shell). From top left to bottom right: spherical sampling – sparse single shell, spherical sampling – dense single shell, Cartesian sampling – dense multi grid, spherical sampling – dense multi shell.

Figure 2: Exemplary signal visualization. Left: Measured signal in q-space for two fiber directions. Right: Resulting dODF with visualized fiber directions.
Reconstruction

Since the Nyquist-Shannon sampling theorem [4] has to be taken into account while reconstructing the q-space, a perfect reconstruction would require an infinitely dense sampled q-space, which is technically not possible. Therefore, many different reconstruction approaches have been published to face this problem.

The presentation of each algorithm comprises different parts. First, the algorithm is conceptually presented, followed by main arguments for and against this algorithm. In the end of each description, we refer to further advanced versions of the basic algorithm.

Since a detailed explanation of each algorithm and its extensions would go beyond the scope of this chapter, only an overview about the basic and well-established concepts is given. Further details can be found in the reference for each method.

Moreover, this section is distributed into two subsections. The first subsection covers the well-established reconstruction algorithms, while the second subsection present different global enhancement approaches.

Reconstruction Algorithm

Diffusion Spectrum Imaging

The Diffusion Spectrum Imaging (DSI) was introduced by Wedeen et al. [5, 6]. It represents the most simple and intuitive approach of reconstructing, which requires a dense sampling of the q-space. After sampling, an IFT is applied to the data in order to recover the dODF in each voxel. A normal DSI sequence samples up to 515 gradients \( G \) on a Cartesian grid in q-space, with a b-value till \( b = 20,000 \, \frac{s}{mm^2} \) (see Figure 2). Due to this dense sampling scheme, the dODF is very accurate, but leads to high acquisition times. In order to use DSI for in-vivo measurements, the resolution must be limited, to decrease the scanning time. This leads to artifacts in the interpolation. Additionally, state-of-the-art hardware is required to measure such high gradient strength.

Pro:
- Achieves a high angular accuracy.

Contra:
- Requires high b-values up to \( 20,000 \, \frac{s}{mm^2} \).
- Requires up to 515 different gradient directions.
  - Resulting in very long acquisition times.
- If the resolution is decreased to gain a speed-up, interpolation is required.
  - Resulting in a blurring effect.

**Enhancements:**
- Employment of a spherical sampling scheme.
- Applying of compressed sensing [7].

**Diffusion Orientation Transform**

The diffusion orientation transform (DOT) [8] was introduced to compute a probability function of the dODF analytically. For this reason, the diffusion decay is assumed to be mono-exponential with the gradient strength $b$. Therefore, in the acquisition process only a single shell in q-space has to be measured (see Figure 2). Only the direction $u$ of $G$ has to be modified and the gradient strength $b$ remains constant. After measuring the signal in q-space, the radial part of the FT is analytically evaluated using spherical harmonics (SH).

**Pro:**
- Achieves an almost as good angular accuracy as DSI.

**Contra:**
- Requires still an infinite sampling of a single shell in q-space.
  - Resulting in long acquisition times.
- Assumption of a mono-exponential signal decay.

**Q-Ball Imaging**

An even simpler way to reconstruct the dODF is Q-Ball Imaging (QBI). It was introduced by Tuch in 2004 and is a commonly used method [9]. The basic idea is that the ODF is assumed to be a radial projection of a measured sphere in q-space. For this reason QBI utilizes the Funk-Radon-Transform (FRT), which directly maps a sphere of the q-space onto a target sphere that represents the dODF. In addition, only a single shell has to be measured in q-space like in the previously explained DOT reconstruction algorithm.

Using a SH basis as target sphere structure for the FRT, the algorithm is simplified and the solution of the FRT can be found analytically [10].

**Pro:**
- Easy and fast analytical solution.
• Only a single shell with a medium number of gradients has to be measured in q-space.
  ➔ Resulting in low acquisition times.
• Robust to noise.

Contra:
• SH instable if the chosen SH-order is too high.
  ➔ Resulting in a decreased angular accuracy.

Enhancements:
• SH basis with regularization term (Laplace-Beltrami coefficient) [9].
• Employment of a sharpening deconvolution transform [10].

Persistent Angular Structure
The persistent angular structure algorithm (PAS) was introduced by Jansons and Alexander in 2003 [12]. Their algorithm projects the normalized diffusion measurements to a sphere, which is assumed to have a similar structure as the fODF. This projection was first computed using a maximum-entropy parameterization, while a second approach replaced it with a combination of linear bases [13]. It has been shown that their first approach has much sharper results and is less smooth, while on the other hand non-linear fitting has to be used to find the maximum entropy, which increases the computational effort.

Pro:
• Achieves a high angular accuracy.

Contra:
• Requires non-linear optimization for entropy maximization.
  ➔ Resulting in long computation times.
• Requires a dense sampling of a single or multiple spheres in q-space.
  ➔ Resulting in long acquisition times.

Enhancements:
• Maximum-entropy representation replaced by a linear basis representation [13].
  ➔ Resulting in a faster computation, but lower angular accuracy.
**Spherical Deconvolution**

Spherical deconvolution (SD) was introduce by Tournier et al. and is based on the idea that each diffusion signal contains a symmetric single-fiber response function $R$, which is constant in the whole brain [14]. Under this assumption, the measured signal $S$ can then be represented as a convolution of the voxel specific fODF $F$ with a constant function $R$. Taking this into account, the signal $S$ can be represented via

$$S(u) = \int_{|w|=1} R(u \ast w)F(w) \, dw,$$

where $u$ and $w$ denote normalized gradient vectors. To estimate the response function $R$, voxels containing a single fiber (for example the corpus callosum) are utilized. After the determination of $R$, the fODF $F$ is derived using a spherical deconvolution. This can be simplified by estimating spherical harmonic bases for $F$ and rotational harmonic bases for $R$ [15].

**Pro:**
- Directly estimates the fODF.
- Deconvolution results in a very short computation time.

**Contra:**
- Achieves a medium angular accuracy.
- Not robust to noise.
- Employs a model-dependent response function.

**Enhancements:**
- Employment of a constrained spherical deconvolution [16].
- Employment of a truncated SH basis [17].
- Employment of a multi-shell approach [18].

**Diffusion Tensor Imaging**

One of the earliest and one of the most simple reconstruction algorithms is the diffusion tensor imaging reconstruction algorithm (DTI) [1]. This algorithm assumes that the signal in every voxel can be modeled using a three-dimensional Gaussian distribution and represented as a diffusion tensor. Due to the fact, that this reconstruction method is blind in case of complex fiber structures, such as fiber crossings, because of its assumption that each voxel can be represented as one tensor, it should be enhanced to the Multi-Diffusion-Tensor model (MDT) [19].
A MDT combines multiple diffusion tensors to detect complex fiber structures and distinguish between different fibers. With this, the signal $S(u)$ can be represented as

$$S(u) = S_0 \sum_{i=1}^{N} f_i e^{-b G D_i G}$$

where $N$ is the number of Tensors, $u$ the applied gradient direction of $G$, $S_0$ is a non-diffusion-weighted signal, $b$ is the chosen gradient strength, $f_i$ is the corresponding volume fraction and $D_i$ is the estimated symmetric 3x3 tensor with six degrees of freedom.

For MDT prior knowledge about the expected number $N$ of different directions in each voxel is required. In addition, each MDT needs a non-linear algorithm to fit $N$ tensors.

Since each tensor is assumed to be symmetric, a MDT model consists $6 \times N$ degrees of freedom. Therefore, at least $6 \times N$ diffusion-weighted images and an additional baseline image without a gradient direction ($b = 0 \text{ } \frac{s}{\text{mm}^2}$) is required. Another $N-1$ gradients are required to define the weight of each tensor. In order to increase the accuracy, more measurements can be used to refine the signal.

**Pro:**
- Achieves a good angular accuracy if $N$ is known.

**Contra:**
- Non-linear optimization required.  
  - Resulting in long computation times.
- Requires a-priori knowledge about the number of tensors $N$.

**Enhancements:**
- Employment of a SH voxel classification based on SD to estimate $N$ [16].
- Simplify the MDT to the Ball-and-Stick model [20].
- Simplify the MDT to the CHARMED Model [21].
**Ball-and-Stick Model**

The Ball-and-Stick model (BS) is a simplified version of the previously described MDT model [20]. It assumes that each signal can be divided into one isotropic ball-part and $N$ anisotropic sticks, leading to $N+1$ compartments. Taking Equation (2) into account, the first eigenvalue $\lambda_1$ for every diffusion tensor $D_i$ is assumed to be equal. Considering the second and third eigenvalue $\lambda_2$ and $\lambda_3$, two cases have to be distinguished. For the ball-case each eigenvalue is equal to ensure that it is isotropic ($\lambda_1 = \lambda_2 = \lambda_3$), while each stick is considered as perfect linear ($\lambda_2 = \lambda_3 = 0$). In order to represent the signal each fiber direction is fitted to a single stick.

*Pro:*
- Achieves a good angular accuracy if $N$ is known.

*Contra:*
- Model is a simplified approximation of the MDT.
- Non-linear optimization is required.
  - Resulting in long computing times.
- Requires a-priori knowledge about the number of tensors $N$.

**Composite Hindered and Restricted Model of Diffusion**

The Composite Hindered and Restricted Model of Diffusion (CHARMED) model [21] assumes that each type of tissue affects the diffusion of water molecules in a voxel. In case of white matter the diffusion signal is restricted, while elsewhere a “hindered” diffusion takes place. Considering a voxel the signal is constructed using a weighted combination of both types. In their first approach Assaf et al. represented the restricted diffusion as a cylinder and the “hindered” diffusion with an anisotropic Gaussian model. Similar to the BS model this model approach has to be fitted to the signal, which requires a non-linear optimization.

*Pro:*
- Achieves a good angular accuracy if $N$ is known.

*Contra:*
- Model is a simplified approximation of the MDT.
- Non-linear optimization is required.
  - Resulting in long computing times.
- Requires a-priori knowledge about the number of tensors $N$. 
Global Enhancements

Dictionary based Enhancements
A new kind of reconstruction algorithm enhancements is the dictionary based approach. Their basic idea is the applicability of compressed sensing (CS) [22], which denotes that the signal is sparsely representable through a linear combination of different basis functions. These bases are taken from a dictionary that is designed in such a way that it covers all possible shape variations of an ODF. Using this dictionary the required gradient measurements and thus the measurement time can be reduced. The refined signal is then combined with a state of the art approach like QBI to estimate the ODF.

**Spherical Ridgelet Method**
Michailovich and Rathi [23] extended the SH basis and approximated the diffusion signal using a Spherical Ridgelet (SR) generating function, based on the Gauss-Weierstrass kernel that is transformed using the Funk-Radon transform. The resulting basis function is able to represent signals that are defined on a single sphere in q-space.

*Pro:*
- Robust to noise.
- Reduces the required number of gradients.
  - ➔ Resulting in a low acquisition time.

*Contra:*
- Model-based approximation of the signal
  - ➔ Resulting in an decreased angular accuracy if model is chosen wrong

*Enhancements:*
- Employment of a Multi Shell SR (Sparse multi-shell diffusion imaging) [24].

**Spherical Polar Fourier Method**
In case of the Spherical Polar Fourier reconstruction (SPF) [25] the dictionary is constructed using a Spherical Polar Fourier basis. This basis can be separated into a radial and an angular part. The angular structure is represented via a Spherical Harmonic basis, while the radial part is assumed to have a Gaussian-like behavior. Therefore, this approach uses the Gauss-Laguerre basis, which is able to detect anisotropic fiber configurations.

*Pro:*
- Robust to noise.
- Reduces the required number of gradients.
  ➔ Resulting in a low acquisition time.

Contra:
- Model-based approximation of the signal
  ➔ Resulting in an decreased angular accuracy if model is chosen wrong

**Combination Enhancements**

Considering each previously described method, it can be seen that each reconstruction algorithm has its pros and cons. To extend their limits and face their disadvantages these basic concept can be combined to create a more robust reconstruction algorithm. In the following an exemplary method is described.

**Spherical Deconvolution combined with the Ball-and-Stick model**

This reconstruction algorithm [17] combines the very fast SD method with the very accurate BS model. The signal is therefore previously deconvoluted based on a Spherical Harmonic basis, which separates the fODF from the signal. Afterwards the BS model is fitted, while the previously required number of directions is estimated via the derived maxima of the fODF. This significantly increases the fitting process of the BS model.

**Pro:**
- A-priori knowledge achieved by SD speeds up the non-linear optimization.
- BS model achieves a good angular accuracy if \( N \) is known.

**Contra:**
- Requires a-priori knowledge about the number of tensors \( N \).
- Not robust to noise.
- Model-depedened response function.

**Enhancements:**
- Combination with SR [26].
**Result and Discussion**

The possibility to reconstruct and to visualize individual neural pathways non-invasively using diffusion imaging provides new opportunities for neurosurgical, neurophysiological and neuropsychological scenarios. Since each presented algorithm targets a different reconstruction problem, it would be complex if not impossible, to compare them quantitatively. Therefore, the following discussion will present qualitative results only. Moreover, it should be kept in mind that only the basic concepts are presented. In case of enhanced reconstruction approaches, different results may occur.

**Summary**

The summary that is given in Table 1 ranks each algorithm qualitatively into four different categories. The first category represents the influence of signal noise on the reconstruction. The second category displays the required number of gradients, while the third category shows the impact on the computation time. The last category expresses the angular error that can be expected. The ranking is derived based on [27, 28].

<table>
<thead>
<tr>
<th>Method</th>
<th>Noise</th>
<th>Gradients</th>
<th>Time</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>MDT</td>
<td>o</td>
<td>++</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>Ball-and-Stick</td>
<td>o</td>
<td>++</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>CHARMED</td>
<td>o</td>
<td>++</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>SD</td>
<td>-</td>
<td>o</td>
<td>++</td>
<td>0</td>
</tr>
<tr>
<td>SD + BS</td>
<td>o</td>
<td>o</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>QBI</td>
<td>o</td>
<td>o</td>
<td>o</td>
<td>o</td>
</tr>
<tr>
<td>QBI (SH)</td>
<td>o</td>
<td>o</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>QBI (SR/SPF)</td>
<td>+</td>
<td>+</td>
<td>o</td>
<td>-</td>
</tr>
<tr>
<td>DSI</td>
<td>--</td>
<td>--</td>
<td>o</td>
<td>++</td>
</tr>
<tr>
<td>DOT</td>
<td>o</td>
<td>-</td>
<td>o</td>
<td>+</td>
</tr>
<tr>
<td>PAS</td>
<td>0</td>
<td>--</td>
<td>--</td>
<td>+</td>
</tr>
</tbody>
</table>

**Discussion**

Drawing a general conclusion is nearly impossible because of the broad range of pros and cons.
Nevertheless, we will point out specific algorithms considering the four main categories shown in Table 1. In the end of this section a short guideline for clinical application is provided.

The impact of noise
The first column of Table 1 represents the impact of noise on the reconstruction. Here, several effects may appear while reconstructing the signal. Noise results in a blurring effect in the signal, which may add new local maxima or shift existing ones. If the signal noise is rather low, the reconstruction ODF will only be shifted or blurred. This leads to a deflected fiber direction and results in a higher angular error or a decreased accuracy, respectively. However, a new peak, caused by high noise in the signal, might result in a completely wrong detected direction.

Comparison shows that DSI appears to be most susceptible to noise. That is why further publications improved its robustness [28]. SD performs slightly better, but is still more susceptible to noise then the other algorithms, why it was further improved via a non-negative constrained approach [18].

The best qualitative result may be achieved by a dictionary based methods QBI (SR/SPF) in case of signals with very high noise [26, 28]. However, such high noise levels are not likely to occur in a MRI system.

Apart from DSI, SD and QBI (SR/SPF) all other models are similarly robust to noise, while [28] showed that methods that use a higher number of gradients are slightly more stable and compensate measurement noise.

Required number of gradients
When considering the minimally required number of gradients to achieve a reliable accuracy, clear differences between the proposed approaches can be observed.

The best result is achieved by the MDT models. To define a MDT model with $N$ estimated tensors usually $6 \times N + N - 1$ different gradient directions are required, which are normalized by an additional non-diffusion image. Any additional measurement refines the fitting process.

Due to the fact that a SH approximation is instable for less gradient directions, the methods based on SH perform worse. A possibility to improve is employment of a truncated regularized SH version to reconstruct the ODF [10]. A better method is the reconstruction achieved by
dictionary based methods. These methods are based on the theory of compressed sensing, which was developed for sparsely sampled signals. Under the condition that prior knowledge about the signal is available, it can be reconstructed from sparse samples.

As expected, DSI, PAS and DOT performed worst, due to the fact that they require a completely sampled sphere or a perfectly sampled q-space, respectively.

**Estimated reconstruction time**

Considering the computational effort, major differences between the proposed methods can be observed.

The fastest method is the SD reconstruction method followed by QBI using a SH basis. Comparable results can be achieved by combing SD with the BS model. Since all these models are based on a SH basis, the analytical fitting process is very fast, while the accuracy is, however, decreased.

PAS and MDT models have the highest computing time, which is due to the non-linear optimization problem that has to be solved.

**Achievable angular accuracy**

One of the most important criteria is the achievable angular accuracy.

The best angular accuracy is achieved by DSI, which is followed by the DOT and PAS algorithm. This is due to the detailed sampling of the q-space, which increases the achievable angular resolution.

On the other hand, the lowest angular accuracy is achieved by QBI in combination with a dictionary based method or SH, because of an approximation error.

**Conclusion**

In summary, every reconstruction algorithm has its pros and cons and none of them outperforms the others in each category. Through the fact that this book is intend to provide a state of the art for clinical researchers in particular, a short guideline is given to identify an optimal reconstruction method for different clinical research scenarios:
• If the field of research offers the possibility to measure ex-vivo tissues and allows long acquisition times, the highest accuracy is achieved using DSI.

• For the case of limited acquisition times a combinative method such as SD + BS [17] appears to be a good compromise between acquisition time and achievable accuracy.

• Normal DTI (or MDT with $N = 1$) should never be used, if more than one fiber direction may occur.

• A good approach to reduce the acquisition time or the number of gradients is the combination of well-established methods such as QBI or the BS with dictionary based methods [23, 26].

• Each algorithm shows a similar susceptibility to noise, i.e. an additional, previously applied noise reduction is useful to increase the angular accuracy.

All in all it should be noted that this is a qualitative comparison considering the pros and cons of the basic concepts only.
References


